SENR/NRPy+: Numerical Relativity in Singular, Curvilinear Coordinate Systems

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In collaboration with

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Thomas W. Baumgarte

Phys. Rev. D 97, 064036 (2018)

http://tinyurl.com/SENRcode

Beyond Black Holes in a Box:
Exploiting Near-Symmetries in NR with Singular Coords

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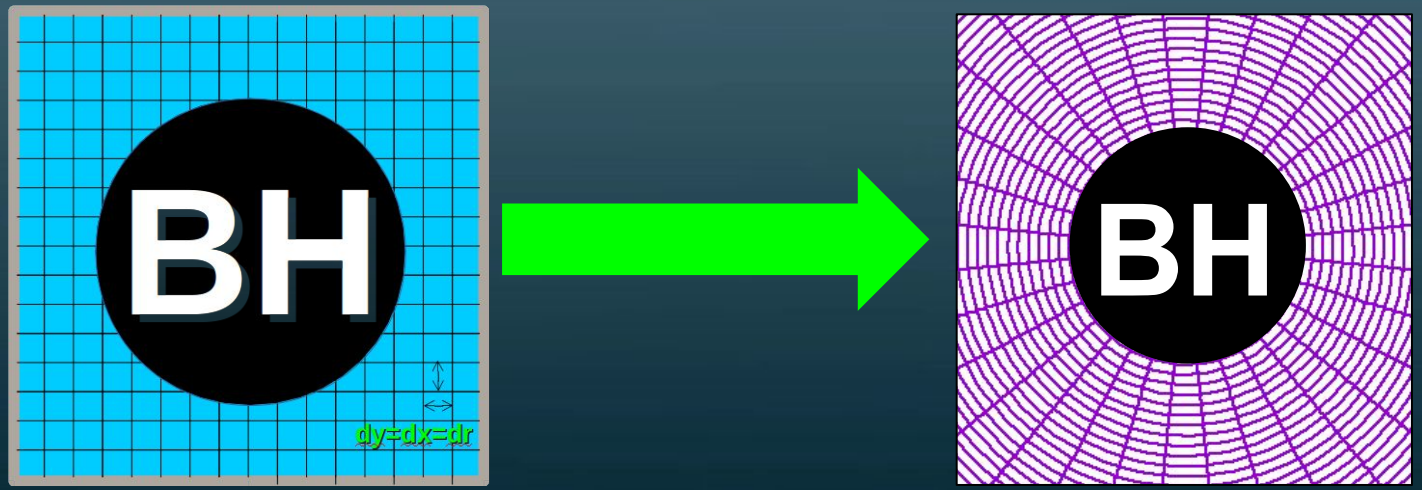

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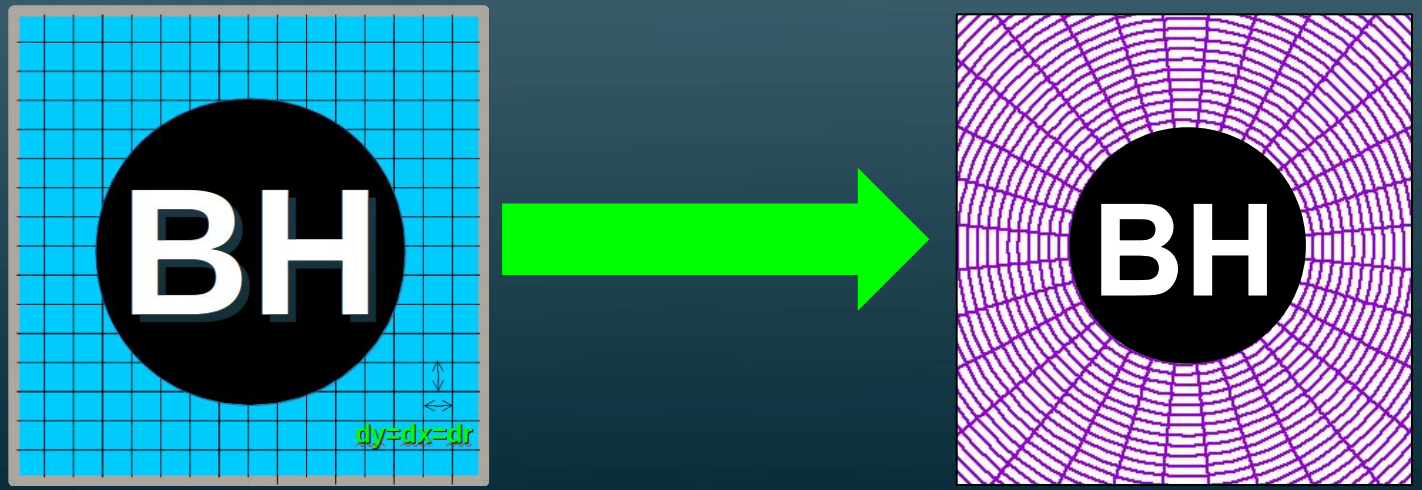

Exploit near-symmetries → Minimize sampling points

BHB Idea: Spherical coord. sampling near each BH & far away

BoE: ~180x fewer points for BHBs than Cartesian AMR

Supercomputer → Consumer-grade desktop!

Toward Black Hole Binaries on the Desktop

1. Extend BSSN-in-spherical-coordinates technique to broader class of static, singular coordinates
   - Implement this technique in new SENR/NRPy+ code

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   - First moving puncture BHB evolutions in singular curvilinear coords
   - First BHB evolutions in corotating frame, without excision

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Addressing Issues with Singular Coordinates

1. **Tensor components can be singular ($\to 0$ or ∞) at coord singularities**
   - Singular pieces are multiplicative and known analytically
   - Scale out singular pieces & handle spatial derivs analytically
   - Promote rescaled tensors to evolved quantities

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2. **Divergent multiplicative terms in RHSs of equations**
   - E.g., 1D scalar wave equation:

$$\partial_t^2 u = \partial_r^2 u + \frac{2}{r}\partial_r u$$

   - 2/*r* term "stiffens" the equation
   - Even with cell-centered grids, RK2 timestepping is unstable
     - Can use PIRK2 (original formulation), but
     - *Ordinary* RK4 works just fine in 3+1 NR (discovered later)

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***Net result*: Stability & convergence properties *on par* with Cartesian grids**

SENR/NRPy+: BSSN in Singular Coordinate Systems

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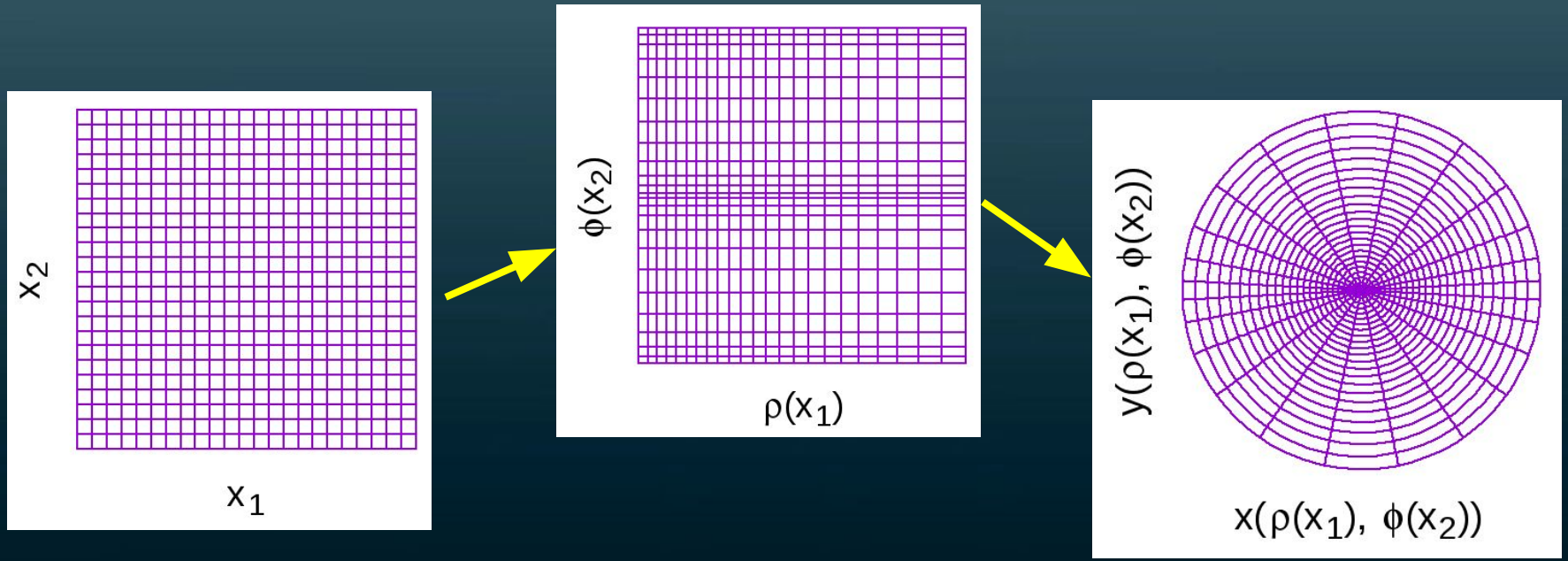
- **Our work** *Phys. Rev. D 97, 064036 (2018), arXiv: 1712.07658*
  - Generalize BSSN moving puncture formalism, to much broader class of singular coordinate systems



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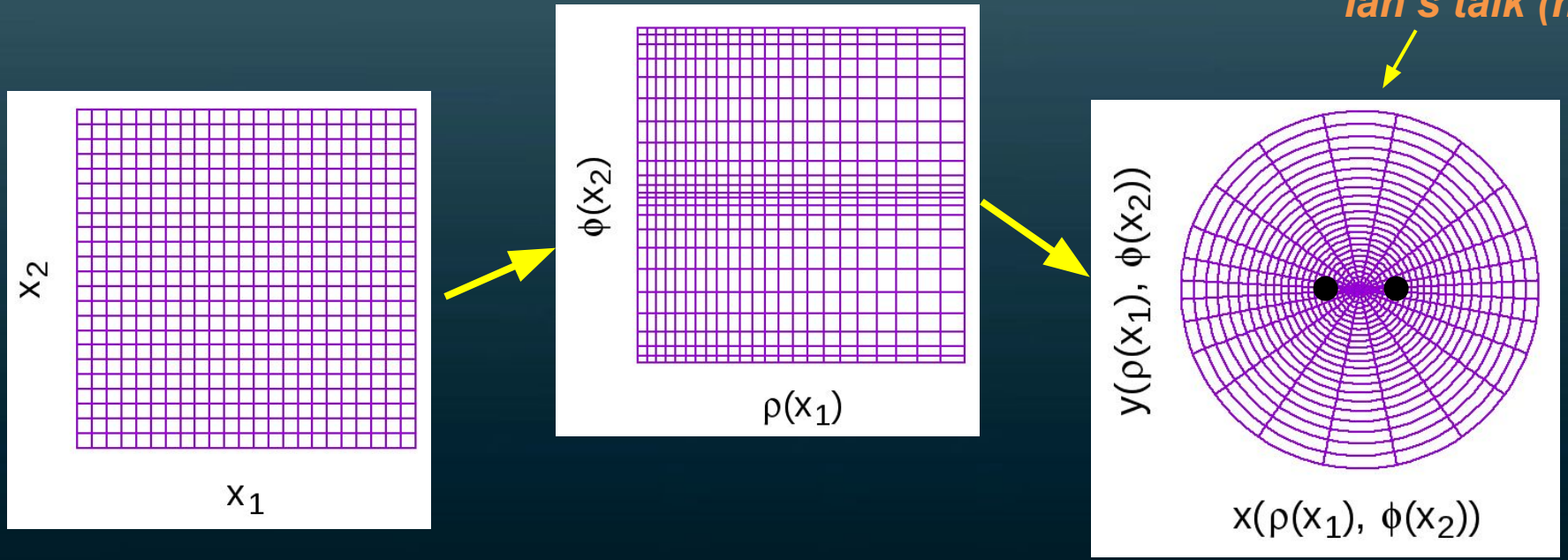
Ian's talk (next)



SENR/NRPy+: Code Overview

http://tinyurl.com/SENRcode

- **NRPy+**: “Python-based code gen for NR… and beyond!”
  - Input: Python expressions in Einstein notation
  - Output: Highly efficient **C code** (AVX intrinsics + FD stencil capable)
  - Implements BSSN in spherical-like, cylindrical-like, and Cartesian-like coords
- **SENR**: “Simple, Efficient Numerical Relativity code”
  - Everything an OpenMP-parallelized NR code needs except equations
    - NRPy+ provides needed C code
    - SENR: RK4 timestepping, boundary condition routines, etc.
  - In spherical coords, results agree to roundoff with Baumgarte *et al* code
- **Both:** Open source (BSD-licensed), designed for user friendliness

SENR/NRPy+: Code Validation

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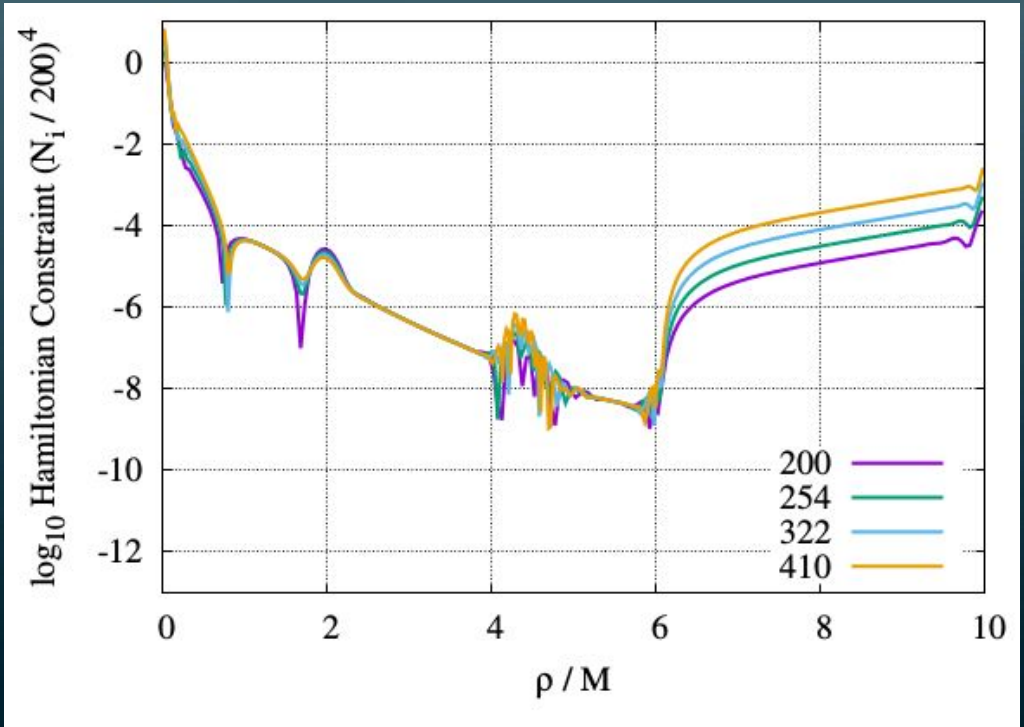
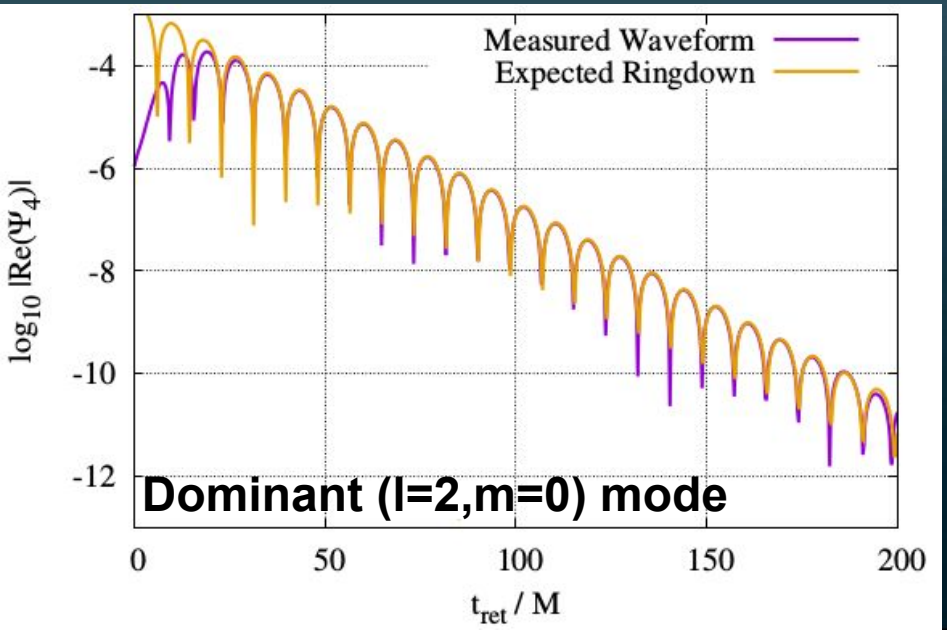
- Black hole simulation
  - Wormhole initial data
  - Cylindrical coordinates
  - Fourth-order finite differencing
- Excellent convergence
  - at t = 5M, in region unaffected by outer boundary (at r=10M)



SENR/NRPy+:
BH Spectroscopy from Head-on BH Collision

- Dual black hole simulation
  - Brill-Lindquist initial data
  - Moving puncture gauge
  - Sinh-spherical coordinates
  - Moderate resolution



- BH perturbation theory prediction
  - **Agreement to ~ 7 decades!**

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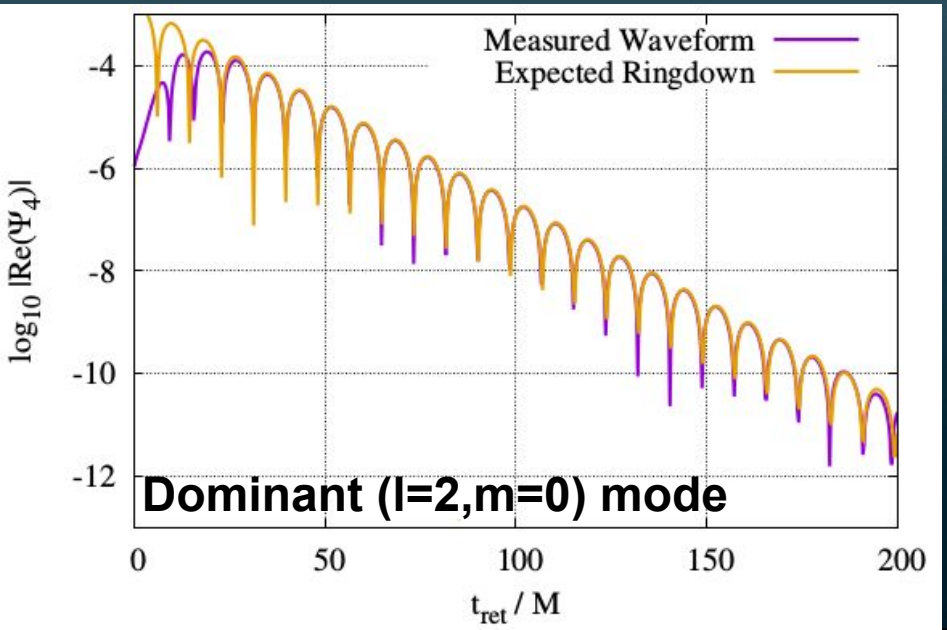
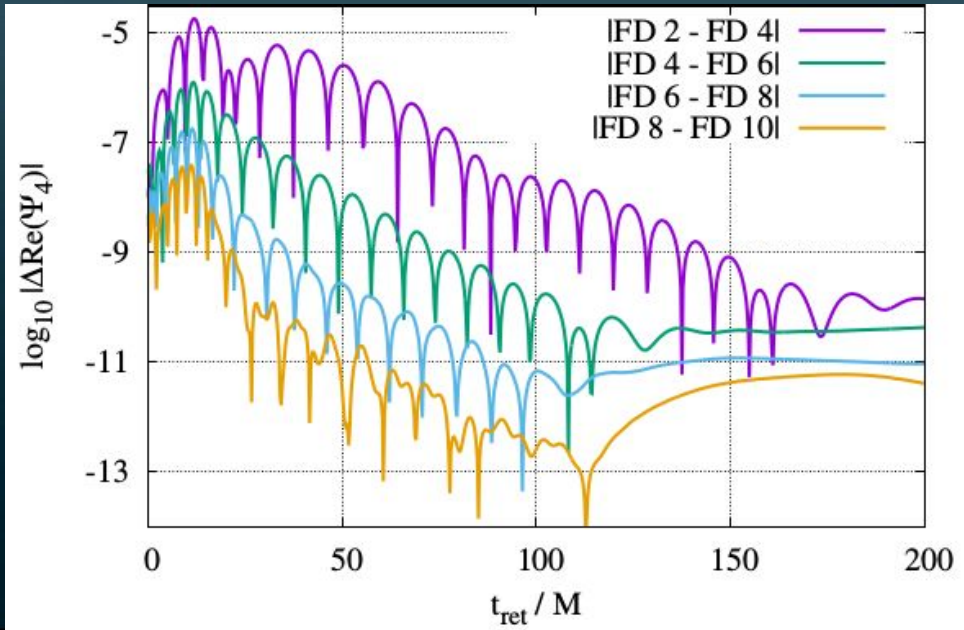
- Increase FD order, grids fixed
  - Nearly exp. convergence in WFs





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Additional Slides

SENR/NRPy+: Nearly Exponential Convergence

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- When truncation error dominates, finite difference error should scale as

$$\left|C_n u^{(n+1)}(\xi)\right| (\Delta x)^n$$

- If grid is uniform,

$$C_n \sim 1/4^n$$

- Thus *so long as* $u^{(n+1)}(\xi)$ *is bounded,* exponential convergence with *fixed* Δx but increasing FD order *n* can be expected.
- We find that although this function is not bounded in our cases, we observe *nearly* exponential convergence at the FD orders chosen